

1 Non-localized Natural Deduction

Introduction Rules

$$\frac{X \quad Y}{X \wedge Y} \wedge I$$

$$\frac{\begin{array}{c} \overline{X}^x \\ \vdots \\ Y \end{array}}{X \rightarrow Y} \rightarrow I_x$$

$$\frac{X}{X \vee Y} \vee I^L \quad \frac{Y}{X \vee Y} \vee I^R$$

$$\frac{}{\perp} \perp I$$

none

Elimination Rules

$$\frac{X \wedge Y}{X} \wedge E^L \quad \frac{X \wedge Y}{Y} \wedge E^R$$

$$\frac{X \rightarrow Y \quad X}{Y} \rightarrow E$$

$$\frac{\begin{array}{c} \overline{X}^x \quad \overline{Y}^y \\ \vdots \quad \vdots \\ X \vee Y \quad Z \quad Z \end{array}}{Z} \vee E_{x,y}$$

none

$$\frac{\perp}{\varphi} \perp E$$

2 Localized Natural Deduction

Introduction Rules

$$\frac{\Gamma \vdash \mathcal{X} : X \quad \Gamma \vdash \mathcal{Y} : Y}{\Gamma \vdash \langle \mathcal{X}, \mathcal{Y} \rangle : X \wedge Y} \wedge I$$

$$\frac{\Gamma, x : \varphi \vdash \mathcal{Y} : Y}{\Gamma \vdash \lambda x. \mathcal{Y} : X \rightarrow Y} \rightarrow I_x$$

$$\frac{\Gamma \vdash \mathcal{X} : X}{\Gamma \vdash \text{inl } \mathcal{X} : X \vee Y} \vee I^L$$

$$\frac{\Gamma \vdash \mathcal{Y} : Y}{\Gamma \vdash \text{inr } \mathcal{Y} : X \vee Y} \vee I^R$$

$$\frac{}{\Gamma \vdash \text{tt} : \top} \top I$$

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Elimination Rules

$$\frac{\Gamma \vdash \mathcal{P} : X \wedge Y}{\Gamma \vdash \text{fst } \mathcal{P} : X} \wedge E^L$$

$$\frac{\Gamma \vdash \mathcal{P} : X \wedge Y}{\Gamma \vdash \text{snd } \mathcal{P} : Y} \wedge E^R$$

$$\frac{\Gamma \vdash \mathcal{F} : X \rightarrow Y \quad \Gamma \vdash \mathcal{X} : X}{\Gamma \vdash \mathcal{F}\mathcal{X} : Y} \rightarrow E$$

$$\frac{\Gamma \vdash \mathcal{D} : X \vee Y \quad \Gamma, x : X \vdash \mathcal{Z} : Z \quad \Gamma, y : Y \vdash \mathcal{Z}' : Z}{\Gamma \vdash \text{case } \mathcal{D} \text{ of } \{ \text{inl } x \mapsto \mathcal{Z} ; \text{inr } y \mapsto \mathcal{Z}' \} : Z} \vee E_{x,y}$$

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$$\frac{\Gamma \vdash \mathcal{L} : \perp}{\Gamma \vdash \perp \text{elim } \mathcal{L} : \varphi} \perp E$$